INTERACTION OF QUASIUNIFORM MAGNETIC FIELD WITH ENSEMBLE OF MHD OSCILLATIONS

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It is shown that inhomogeneous MHD turbulence in a cold plasma manifests itself as an inhomogeneous diamagnet. An equation describing the evolution of the regular component of the magnetic field is derived and a formula obtained whereby the coefficient of turbulent magnetic field diffusion can be estimated. Estimates are made which indicate that this is an efficient mechanism for the decay of the magnetic field of a sunspot.

It is well known that turbulence in a nongyrotropic medium causes anomalous diffusion of a large-scale magnetic field. Diffusion coefficients are obtained in [1, 2] that differ substantially from the ohmic diffusion coefficient ν if $Re_m \gg 1$ (Re_m is the magnetic Reynolds number). The effect of the magnetic field on the turbulent disturbances was assumed in these papers to be negligible.

In the present paper we investigate the opposite limiting case, i.e., the magnetic field is assumed to have a strong effect on the turbulent disturbances. The turbulent motion, if its total energy is small compared with the energy of the plasma, can accordingly be represented as a superposition of Alfvén and magnetoacoustic waves. It is natural to assume that $L \gg l$, where L is the characteristic scale of the inhomogeneity of the magnetic field, and l is the characteristic scale of the turbulence. This sort of interaction between turbulence and a magnetic field can be observed in sunspots and also in experiments with laboratory plasmas.

1. Basic Equations. Equation for the Magnetic Field

We utilize the following assumptions: a) the turbulence is stationary; b) $\beta \ll 1$ (the cold plasma approximation); c) $Re_m \gg 1$; d) $\langle \mathbf{v} \rangle = 0$ (the angular brackets denote an average over an ensemble), i.e., there are no macroscopic motions in the plasma. The form of the spectral function of MHD oscillations is known [3].

Low-frequency oscillations will then be described by the following system of equations of magnetohydrodynamics:

$$\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} = -(1/\rho)\nabla p + (1/4\pi\rho)[\mathrm{rot}\,\mathbf{H},\ \mathbf{H}]; \tag{1.1}$$

$$\partial \rho/\partial t + \mathrm{div}(\rho \mathbf{v}) = 0; \tag{1.2}$$

$$\partial \mathbf{H}/\partial t = \mathrm{rot}[\mathbf{v},\ \mathbf{H}] + \nu \Delta \mathbf{H}, \tag{1.3}$$

where ρ is the density; p is the pressure; $\mathbf{v}$ is the velocity; H is the magnetic field intensity; $\nu = c^2/4\pi\sigma$; σ is the conductivity; and c is the velocity of light.

We obtain an equation connecting the magnetic field with the turbulent velocity field from (1.3). To this end we represent the magnetic field in the form

$$\mathbf{H}(\mathbf{r},\ t) = \mathbf{H}^{\mathrm{R}}(\mathbf{r},\ t) + \mathbf{H}^{\mathrm{T}}(\mathbf{r},\ t),$$

where $\mathbf{H}^{\mathrm{R}}$ and $\mathbf{H}^{\mathrm{T}}$ are, respectively, the regular and turbulent components of the magnetic field, and

$$\langle \mathbf{H}(\mathbf{r},\ t)\rangle = \mathbf{H}^{\mathrm{R}}(\mathbf{r},\ t), \quad \langle \mathbf{H}^{\mathrm{T}}\rangle = 0.$$

We average Eq. (1.3) over an ensemble and obtain an equation describing the evolution of the regular magnetic

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field

$$\partial \mathbf{H}^R/\partial t = \mathrm{rot}\,\langle\, [\mathbf{v}\mathbf{H}^\tau]\,\rangle + \nu\Delta \mathbf{H}^R. \tag{1.4}$$

In this expression the first term on the right side is unknown. We find it by utilizing the equation for the turbulent component

$$\partial \mathbf{H}^\tau/\partial t = \mathrm{rot}\,[\mathbf{v}\mathbf{H}^R] + \nu\Delta \mathbf{H}^\tau. \tag{1.5}$$

The diffusion term in (1.5) can be neglected as $t_D \gg t_F$, where t_D is the characteristic ohmic diffusion time and t_F is the characteristic time of the magnetic field fluctuations. The turbulent component of the magnetic field can be represented in the form

$$\mathbf{H}^\tau(\mathbf{r}, t) = \int\limits_0^t \mathrm{rot}\,[\mathbf{v}(\mathbf{r}, t_1)\,\mathbf{H}^R(\mathbf{r}, t_1)]\,dt_1. \tag{1.6}$$

Inserting (1.6) into $\langle [\mathbf{v}\mathbf{H}^T]\rangle$ and carrying out the averaging, we obtain

$$\langle[\mathbf{v}\mathbf{H}^\tau]\rangle = \varepsilon_{ijk}\int\limits_0^t \left\{\frac{\partial}{\partial x_l}(H_k(\mathbf{r}, t_1)\,V_{jl}(\mathbf{r}, 0, s) - (\mathbf{H}^R(\mathbf{r}, t_1)\nabla)\,V_{jk}(\mathbf{r}, 0, s)\right\} dt_1, \tag{1.7}$$

where ϵ_{ijk} is the antisymmetric Levi–Civita tensor; $V_{ij}(\mathbf{R}, \rho, s) = \langle v_i(\mathbf{r}_1, t_1)\, v_j(\mathbf{r}_2, t_2)\rangle$; $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$; $\rho = r_1 - r_2$; $s = t_1 - t_2$. The correlation function V_{ij} is weakly dependent on $\mathbf{R}$; it changes appreciably only when $\mathbf{R}$ varies by an amount of the order L. The dependence of the function V_{ij} on the arguments ρ and s describes the local structure of the random velocity field. Since $t_D \gg \tau_C$, where τ_C is the correlation time, we find the integral

$$\int\limits_0^t V_{ij}(\mathbf{R}, \rho, s)\,\mathbf{H}^R(\mathbf{R}, t_1)\,dt_1 \approx \mathbf{H}^R(\mathbf{R}, t)\int\limits_0^\infty V_{ij}(\mathbf{R}, \rho, s)\times$$
$$\times\, ds = \mathbf{H}^R(\mathbf{R}, t)\,B_{ij}(\mathbf{R}, \rho, 0), \tag{1.8}$$

where

$$B_{ij}(\mathbf{R}, \rho, \omega) = \quad V_{ij}(\mathbf{R}, \rho, s)e^{-i\omega s}ds.$$

Utilizing (1.8), we integrate (1.7)

$$\langle[\mathbf{v}\mathbf{H}^\tau]\rangle = \varepsilon_{ijk}\left\{\frac{\partial}{\partial x_l}(H_k^R B_{jl}(\mathbf{r}, 0, 0)) - (\mathbf{H}^R\nabla)\,B_{jk}(\mathbf{r}, 0, 0)\right\}. \tag{1.9}$$

Inserting (1.9) into (1.4) gives an equation for the magnetic field:

$$\frac{\partial H_m^R}{\partial t} = -\,\varepsilon_{mni}\varepsilon_{ijk}\frac{\partial}{\partial x_n}\left\{\frac{\partial}{\partial x_l}(H_k^R B_{jl}(\mathbf{r}, 0, 0)) - (\mathbf{H}^R\nabla)\,B_{jk}(\mathbf{r}, 0, 0)\right\} + \nu\Delta H_m^2. \tag{1.10}$$

2. Determination of Correlation Tensor of MHD Turbulent Velocity Field.

The velocity-field correlation tensor entering into Eq. (1.10) can be expressed in terms of the spectral tensor of this field at zero frequency ω:

$$B_{ij}(\mathbf{r}, 0, 0) = \int T_{ij}(\mathbf{r}, \mathbf{k}, 0)\,dk, \tag{2.1}$$

where $\mathbf{k}$ is the wave vector. We find the spectral tensor from the linearized equations (1.1)-(1.3) for a uniform magnetic field; then, into terms containing H, we introduce the dependence on $\mathbf{r}$ and obtain the quasi-homogeneous spectral tensor of the velocity field.

We represent the density, velocity, and magnetic field in the form

$$\rho = \rho_0 + \rho^{(1)} + \rho^{(2)},\ \mathbf{v} = \mathbf{v}^{(1)} + \mathbf{v}^{(2)},\ \mathbf{H} = \mathbf{H}_0 + \mathbf{h}^{(1)} + \mathbf{h}^{(2)},$$

where

$$\rho_0 \gg \rho^{(1)} \gg \rho^{(2)},\ \mathbf{v}^{(1)} \gg \mathbf{v}^{(2)},\ \mathbf{H}_0 \gg \mathbf{h}^{(1)} \gg \mathbf{h}^{(2)}.$$

We obtain equations for $\rho^{(1)}$, $\mathbf{v}^{(1)}$, $h^{(1)}$ from linearized equations (1.1)-(1.3) in $\mathbf{k}-\omega$ space by neglecting the diffusion term in (1.3) ($t_D \gg t_F$):

$$\omega\rho_0\mathbf{v}_{\mathbf{k},\omega}^{(1)} - c_s\rho_{\mathbf{k},\omega}^{(1)}\mathbf{k} + \frac{1}{4\pi}\left[[\mathbf{k}\mathbf{h}_{\mathbf{k},\omega}^{(1)}]\,\mathbf{H}_0\right] = 0; \tag{2.2}$$

$$\omega\rho_{\mathbf{k},\omega}^{(1)} - \rho_0(\mathbf{k}\mathbf{v}_{\mathbf{k},\omega}^{(1)}) = 0, \tag{2.3}$$

$$\omega \mathbf{h}^{(1)}_{\mathbf{k},\omega} + [\mathbf{k}[\mathbf{v}^{(1)}_{\mathbf{k},\omega}\mathbf{H}_0]) = 0, \tag{2.4}$$

where c_S is the velocity of sound.

Subject to the condition $\beta \ll 1$ the solution of system (2.2)-(2.4) is the familiar dispersion relation for an Alfvén and an accelerated magnetoacoustic wave [4] with the following frequencies, respectively:

$$\omega^{A}_{\mathbf{k}} = \frac{\mathbf{k}\mathbf{H}_0}{\sqrt{4\pi\rho_0}}, \quad \omega^{M}_{\mathbf{k}} = \mathbf{k}c_A,$$

where c_A is the Alfvén velocity.

A study of Eqs. (2.2)-(2.4) shows that $\mathbf{v}^{A(1)}$, $h^{A(1)}$ are perpendicular to H_0 and $\mathbf{k}$ and that $\mathbf{v}^{M(1)}$ is perpendicular to $\mathbf{H}_0$ and lies in the plane of the vectors $\mathbf{k}$ and H_0. We can thus write

$$v^{A(1)}_i = \psi_{\mathbf{k},\omega}\varepsilon_{ijm}k_j\tau_m;$$
$$v^{M(1)}_i = \varphi_{\mathbf{k},\omega}(k_i - (\mathbf{k}\tau)\tau_i),$$

where $\tau = \mathbf{H}_0/H_0$.

For simplicity of presentation we restrict the discussion to interaction between magnetoacoustic waves and set

$$T_{ij}(\mathbf{k},\omega) = T^{M}_{ij}(\mathbf{k},\omega).$$

In the first approximation the velocity amplitudes are independent of time and correspond to the solution for zero interaction between the waves. If the oscillations are developed from random thermal fluctuations, the spectral tensor of the velocity field has the following form in the first approximation:

$$\langle v^{M(1)}_{i\mathbf{k},\omega} v^{M(1)}_{j\mathbf{k}',\omega'}\rangle = \delta(\mathbf{k}+\mathbf{k}')\,\delta(\omega+\omega')\,T^{(1)}_{ij}(\mathbf{k},\omega), \tag{2.5}$$

$$T^{(1)}_{ij}(\mathbf{k},\omega) = \frac{1}{2}\Phi_{\mathbf{k},0}\{k_ik_j + (\mathbf{k}\tau)^2\tau_i\tau_j - (\mathbf{k}\tau)(k_i\tau_j + \tau_ik_j)\}\left(\delta(\omega+\omega^{M}_{\mathbf{k}}) + \delta(\omega-\omega^{M}_{\mathbf{k}})\right),$$

where $\Phi_{\mathbf{k}_0}$ is the spectral function of the MHD oscillations. Accordingly, the tensor $T^{(1)}_{ij}(\mathbf{k},\omega)$ has a δ-type maximum at $\omega = \pm\omega^{M}_{\mathbf{k}}$ and $T^{(1)}_{ij}(\mathbf{k},\omega) = 0$ for $\mathbf{k} \neq 0$ and thus makes no contributionto (2.1). The equations for the second-approximation corrections have the form

$$\partial\rho^{(2)}/\partial t + \rho_0 \operatorname{div}\mathbf{v}^{(2)} = -\operatorname{div}(\rho^{(1)}\mathbf{v}^{(1)});$$
$$\partial\mathbf{h}^{(2)}/\partial t - \operatorname{rot}[\mathbf{v}^{(2)}\mathbf{H}_0] = \operatorname{rot}[\mathbf{v}^{(1)}\mathbf{h}^{(1)}].$$

In the Fourier representation these equations acquire the form

$$\omega\rho^{(2)}_{\mathbf{k},\omega} - \rho_0(\mathbf{k}\mathbf{v}^{(2)}_{\mathbf{k},\omega}) = \int \rho^{(1)}_{\mathbf{k}_2,\omega_2}(\mathbf{k}\mathbf{v}^{(1)}_{\mathbf{k}_1,\omega_1})\,d\lambda; \tag{2.6}$$

$$\omega\mathbf{h}^{(2)}_{\mathbf{k},\omega} - [\mathbf{k}[\mathbf{v}^{(2)}_{\mathbf{k},\omega}\mathbf{H}_0]] = -\int [\mathbf{k}[\mathbf{v}^{(1)}_{\mathbf{k}_1,\omega_1}\mathbf{h}^{(1)}_{\mathbf{k}_2,\omega_2}]]\,d\lambda, \tag{2.7}$$

where

$$d\lambda = \delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2)d\mathbf{k}_1d\mathbf{k}_2d\omega_1d\omega_2.$$

We replace $\rho^{(1)}$ and $\mathbf{h}^{(1)}$ by their values from Eqs. (2.3) and (2.4), and we multiply (2.6) by $\mathbf{H}_0$ and (2.7) by ρ_0. We apply the well-known formula of vector analysis to the second term on the left side of (2.7), subtract (2.6) from (2.7), and obtain

$$\frac{\omega}{\rho_0}(\mathbf{h}^{(2)}_{\mathbf{k},\omega}\rho_0 - \rho^{(2)}_{\mathbf{k},\omega}\mathbf{H}_0) + \mathbf{v}^{(2)}_{\mathbf{k},\omega}(\mathbf{k}\mathbf{H}_0) = \int \frac{\partial\lambda}{\omega^2}\{(\mathbf{k}\mathbf{v}^{(1)}_{\mathbf{k}_1,\omega_1})(\mathbf{k}_2\mathbf{v}^{(1)}_{\mathbf{k}_2,\omega_2})\mathbf{H}_0 - [\mathbf{k}[\mathbf{v}^{(1)}_{\mathbf{k}_1,\omega_1}[\mathbf{k}_2[\mathbf{v}^{(1)}_{\mathbf{k}_2,\omega_2}\mathbf{H}_0]]]]\}. \tag{2.8}$$

We multiply Eq. (2.8) by $v^{(1)}_{i\mathbf{k}',\omega'}$, and average over an ensemble; to the right side of the equality we apply the random phase approximation and obtain as a result

$$\langle v^{(2)}_{i\mathbf{k},\omega} v^{(1)}_{j\mathbf{k}',\omega'}\rangle|_{\omega=0} = 0.$$

We multiply (2.8) by the complex conjugate and average the right and left sides of the equality. Fourth-order moments appear in the right side. We expand them with the aid of the random phase approximation and, having utilized (2.5), we consider the obtained expression for $\omega = 0$:

$$T^{(2)}_{ij}(\mathbf{k},0) = \frac{1}{4}\int \frac{([\mathbf{q}\tau][\mathbf{p}\tau])^2}{c_A(\omega^{M}_{\mathbf{q}})^2}(l_i - (\mathbf{l}\tau)\tau_i)(l_j - (\mathbf{l}\tau)\tau_j)\,\Phi_{\mathbf{q},0}\,\Phi_{\mathbf{p},0}(\delta(p+q) + \delta(p-q)\,d\mathbf{q}, \tag{2.9}$$

where $\mathbf{l} = \mathbf{p} - \mathbf{q}$, $\mathbf{p} = \mathbf{k} - \mathbf{q}$, $\mathbf{q} = \mathbf{k}_1$.

The velocity field spectral tensor (2.9) has the following properties:

$$\tau_i T_{ij}^{(2)}(\mathbf{k}, 0) = 0, \; \tau_j T_{ij}^{(2)}(\mathbf{k}, 0) = 0. \tag{2.10}$$

In the general case, an axisymmetric spectral tensor can be represented in the form

$$T_{ij} = A_1 k_i k_j + A_2 \tau_i \tau_j + A_3 \delta_{ij} + A_4 k_i \tau_j + A_5 \tau_i k_j,$$

where $A_m = A_m(k, \mathbf{k}\tau)$, $m = 1, \ldots, 5$.

The number of unknown functions $A_m(k, \mathbf{k}\tau)$, can be reduced with the aid of (2.10):

$$T_{ij}^{(2)} = A_1 k_i k_j + (A_1(\mathbf{k}\tau)^2 - A_3)\tau_i\tau_j + A_3\delta_{ij} - A_1(\mathbf{k}\tau)(k_i\tau_j + \tau_i k_j),$$

and we find A_1 and A_3 from the following system of equations:

$$k_i k_j T_{ij}^{(2)}(\mathbf{k}, 0) = f_1(\mathbf{k}); \tag{2.11}$$

$$T_{ii}(\mathbf{k}, 0) = f_2(\mathbf{k}), \tag{2.12}$$

where

$$f_1(\mathbf{k}) = \frac{1}{2}\int \frac{([\mathbf{q}\tau][\mathbf{p}\tau])^2}{c_A^3 q}(\mathbf{l}\tau)(\mathbf{k}\tau)\Phi_{\mathbf{q},0}\Phi_{\mathbf{p},0}\delta(p^2 - q^2)\,d\mathbf{q};$$

$$f_2(\mathbf{k}) = \frac{1}{2}\int \frac{([\mathbf{q}\tau][\mathbf{p}\tau])^2}{c_A^3 q}[\mathbf{l}\tau]^2\Phi_{\mathbf{q},0}\Phi_{\mathbf{p},0}\delta(p^2 - q^2)\,d\mathbf{q}.$$

On solving system (2.11), (2.12), we find

$$A_1 = \frac{1}{[\mathbf{k}\tau]^2}\int \frac{([\mathbf{q}\tau][\mathbf{p}\tau])^2}{c_A^3 q}\left\{\frac{(\mathbf{k}\tau)^2(\mathbf{l}\tau)^2}{[\mathbf{k}\tau]^2} - \frac{[\mathbf{l}\tau]^2}{2}\right\}\Phi_{\mathbf{q},0}\Phi_{\mathbf{p},0}\delta(p^2 - q^2)\,d\mathbf{q}; \quad A_3 = \frac{1}{2}\int \frac{([\mathbf{q}\tau][\mathbf{p}\tau])^2}{c_A^3 q}\left\{[\mathbf{l}\tau]^2 - \frac{(\mathbf{k}\tau)^2(\mathbf{l}\tau)^2}{[\mathbf{k}\tau]^2}\right\}\Phi_{\mathbf{q},0}\Phi_{\mathbf{p},0}\delta(p^2 - q^2)\,d\mathbf{q}.$$

We thus have an explicit expression for the spectral tensor of the velocity field $T_{ij}(\mathbf{k}, 0)$ and can accordingly integrate (2.1):

$$\int T_{ij}(k, \mathbf{k}\tau, 0)d\mathbf{k} = (\delta_{ij} - \tau_j\tau_i)(1/2)\int T_{ii}(k, \mathbf{k}\tau, 0)d\mathbf{k}.$$

The quasihomogeneous correlation tensor of the velocity field can then be written in the form

$$B_{ij}(\mathbf{r}, 0, 0) = \varepsilon(\mathbf{r}, t)(\delta_{ij} - \tau_i(\mathbf{r}, t)\tau_j(\mathbf{r}, t)). \tag{2.13}$$

Inserting (2.13) into Eq. (1.10) gives

$$\partial \mathrm{H}^{\mathrm{R}}/\partial t = -\nu \,\mathrm{rot}\,\mathrm{rot}\,\chi \mathrm{H}^{\mathrm{R}}, \tag{2.14}$$

where

$$\chi = 1 + \varepsilon(\mathbf{r}, t)/\nu.$$

It follows from (2.14) that inhomogeneous turbulence manifests itself as an inhomogeneous diamagnet. A similar result was obtained in [5]. Let us suppose that the turbulence ends somewhere, and that in the region where it exists it is homogeneous, i.e., let $\epsilon(\mathbf{r}, t) = \epsilon_0$ for $\mathbf{r} \in Q$ and $\epsilon(\mathbf{r}, t) = \nu$ outside the region Q. The boundary conditions on the magnetic field will then be as follows:

$$\mathbf{H}_{n_1}^{\mathrm{R}} = \mathbf{H}_{n_2}^{\mathrm{R}}; \; \varepsilon_0\mathbf{H}_{t_1}^{\mathrm{R}} = \nu\mathbf{H}_{t_2}^{\mathrm{R}},$$

where $\mathbf{H}_{n_1}^{\mathrm{R}}$, $\mathbf{H}_{n_2}^{\mathrm{R}}$, $\mathbf{H}_{t_1}^{\mathrm{R}}$, $\mathbf{H}_{t_2}^{\mathrm{R}}$ are the normal and tangential components of the magnetic field at the boundary; the index 1 refers to the inside of the surface containing Q and the index 2, to the outside. The magnetic permeability of a diamagnet of this sort $\mu \approx \nu/\epsilon_0$, i.e., let $\mu < 1$ for $\mathrm{Re}_m \gg 1$. The following estimate for ϵ can be written down with the aid of the results of [6]:

$$\varepsilon \approx (1/\pi)(v/c_A)^3 v\lambda_0,$$

where v is the velocity amplitude, and λ_0 is the external scale of the turbulence. If we set $\epsilon(\mathbf{r}, t) = \epsilon_0$, then the characteristic field diffusion time is estimated by

$$t_D \approx L^2/4\pi^2\varepsilon_0 \approx (L^2/4\pi v\lambda_0)(c_A/v)^3.$$

We utilize this formula to estimate the time required to damp the magnetic field of a sunspot; $v \approx 10^5$ cm/sec, $\lambda_0 \approx 6 \cdot 10^7$ cm is the dimension of the granule in the sunspot, and $\rho \approx 2 \cdot 10^{-7}$ g/cm^3. As is known, the diffusion time is determined by the smallest characteristic dimension of the system. In the simplest models the sunspot has the shape of a cylinder. We estimate the depth h of the sunspot from the condition $W_T \simeq W_M$, where

W_T and W_M are, respectively, the energies of the turbulent motion and the magnetic field. Then, for $H = 1000$ G, the sunspot diameter $D \approx 5 \cdot 10^8$ cm, $h \approx 1 \cdot 10^8$ cm; for $H = 2000$ G, $D \approx 9 \cdot 10^8$ cm, $h \approx 2 \cdot 10^8$ cm. Finally, for $H \approx 1000$ G we obtain $t_D \approx 8$ days, and for $H \approx 2000$ G, $t_D \approx 10$ days.

These damping times do not exceed the sunspot lifetime. Accordingly, we can assume that anomalous magnetic field damping as a result of turbulence is one of the factors leading to the disappearance of sunspots.

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